

On the “Attractive Component” to the Free Energy of Interaction between Macroions of Like Charge

The purpose of this Commentary is to clarify some of the recent issues associated with theoretical investigations on macroionic solutions and colloidal suspensions. The DLVO (Derjaguin–Landau–Verwey–Overbeek) pairwise interaction energy^{1,2} is generally used to interpret the physical properties of these charged systems. The DLVO energy is composed of two parts: a short-range van der Waals attraction term and a long-range Coulombic repulsion term. The van der Waals term is significant only when the particles are near contact and is therefore not relevant for the present communication. The DLVO repulsive term between two polyions, denoted by pp and separated by a distance r , is of the form of a screened Debye–Hückel interaction,

$$U_{pp}(r) = \frac{Z_p^2 q_e^2 \exp(\kappa_{DH} a_{pp}) \exp(-\kappa_{DH} r)}{\epsilon(1 + \kappa_{DH} a_{pp})^2 r} \quad (1)$$

where Z_p is the magnitude with sign of the polyion charge, q_e is the magnitude of the electron charge, ϵ is the bulk dielectric constant, a_{pp} is the distance of closest approach (sum of the radii) between two polyions, and κ_{DH} is the Debye–Hückel screening parameter, where the subscript DH indicates that it is a function of *only the added salt*,

$$\kappa_{DH}^2 = 4\pi\lambda_B \sum_{\text{added}} Z_j^2 \langle n_j \rangle \quad (2)$$

where $\langle n_j \rangle$ is the concentration (particles/cm³) of the j th added ion and $\lambda_B = q_e^2/\epsilon kT$ is the Bjerrum length.

Evidence is mounting that indicates that the DLVO theory may not describe the complete picture of these complex macroion systems.^{3–14} In particular, these data indicate the presence of an “apparent long range attraction” between macroions of like charge. There is the direct visual evidence of Ise and co-workers,⁵ who report confocal laser scanning micrographs showing coexisting “void” and “cluster” regions in salt-free dispersions of latex spheres, where the void region extended to a length of 60 μm into the solvent. The cluster regions exhibit a high degree of order as evidenced by ultrasmall-angle X-ray scattering.⁴ The ordered clusters appear to have a dynamic character.^{3,6} In their light-scattering study of the liquid phase of latex spheres, Clark, Ackerson, and

Hurd⁶ describe the dynamics of the underlying ordered regions as:

The scattering becomes characterized by the momentary appearance and disappearance of six-spot patterns reminiscent of the crystalline phase and having the same K_{10} . These fluctuating spot patterns appear with random orientation and persist for a few tens of milliseconds.

The formation of “clumps” of particles is not limited to latex spheres. Dynamic light scattering (DLS) methods indicate the presence of “anomalously slow” dynamic processes that have yet to be explained. Lin, Lee, and Schurr⁷ first reported a catastrophic drop (by a factor of 20) in the apparent diffusion coefficient (D_{app}) of poly(L-lysine) over a *narrow* ionic strength range as the concentration of NaBr was decreased. The abruptness of this “ordinary–extraordinary” (o–e) transition is reflected in their description of the light-scattering intensity:

For example, the 1.0 mg/ml solution exhibited at $\theta = 60^\circ$ a 2.5-fold increase in intensity between 9.1×10^{-4} and 1.1×10^{-3} M NaBr.

The magnitude of this decrease in the value of D_{app} at the transition point was too large to be attributed to the coil–rod transition.^{7,15} The tracer diffusion, or “self-diffusion”, coefficient (D_{self}) does not, however, exhibit such extraordinary behavior. Fluorescence photobleaching recovery experiments on poly(lysine) indicate that D_{self} undergoes a *gradual* decrease in value as one lowers the ionic strength through the o–e transition region. This gradual decrease in D_{self} is what one would expect for a coil expansion.⁸ Entanglement of the molecules is ruled out on the basis of the data of Bruno and Mattice.⁹ In their studies, poly(L-lysine) is labeled with fluorescein or rhodamine B in a mixed system with copolymers of poly(L-lysine-co-L-tryptophan) or poly(L-lysine-co-L-tyrosine). Bruno and Mattice conclude that the interaction between the poly(lysine) in the e-regime does not involve physical contacts but rather occurs at separation distances of 60–100 Å. Polystyrene sulfonate is another system in which D_{app} exhibits a precipitous drop in value over a narrow ionic strength range.¹⁰ Another unexplained phenomenon in DLS studies is the “splitting” of relaxation domains as a function of added salt, such as for tRNA,¹¹ bovine serum albumin,¹¹ quaternized poly(2-vinylpyridine),¹² and mononucleosomal DNA.¹³ Since mononucleosomal DNA is modeled as a cylinder 450 Å in length, the splitting phenomenon for this system cannot be interpreted in terms of entanglements.

Let us now relate the above DLS studies to the DLVO interaction energy. The current models^{16–18} for coupled dynamics in the polyion–electrolyte ion sys-

- (1) Derjaguin, B. V.; Landau, L. *Acta Physicochim.* **1941**, *14*, 633.
- (2) Verwey, E. J. W.; Overbeek, J. Th. G. *Theory of the Stability of Lyophobic Colloids*; Elsevier Publishing Co., Inc.: New York, 1948.
- (3) Kose, A.; Ozaki, M.; Takano, K.; Kobayashi, Y.; Hachisu, S. *J. Colloid Interface Sci.* **1973**, *44*, 330.
- (4) Matsuoka, H.; Kakigami, K.; Ise, N.; Kobayashi, Y.; Machitani, Y.; Kikuchi, T.; Kato, T. *Proc. Natl. Acad. Sci. U.S.A.* **1991**, *88*, 6618.
- (5) Ito, K.; Yoshida, H.; Ise, N. *Science* **1994**, *263*, 66.
- (6) Clark, N. A.; Ackerson, B. J.; Hurd, A. J. *Phys. Rev. Lett.* **1983**, *50*, 1459.
- (7) Lin, S.-C.; Lee, W. I.; Schurr, J. M. *Biopolymers* **1978**, *17*, 1041.
- (8) Zero, K.; Ware, B. R. *J. Chem. Phys.* **1984**, *80*, 1610.
- (9) Bruno, K. R.; Mattice, W. L. *Macromolecules* **1992**, *25*, 327.
- (10) Drifford, M.; Dalbiez, J.-P. *Biopolymers* **1985**, *24*, 1501.
- (11) Patkowski, A.; Gulari, E.; Chu, B. *J. Chem. Phys.* **1980**, *73*, 4770.
- (12) Förster, S.; Schmidt, M.; Antonietti, M. *Polymer* **1990**, *31*, 781.
- (13) Fulmer, A. W.; Benbasat, J. A.; Bloomfield, V. A. *Biopolymers* **1981**, *20*, 1147.
- (14) A more expanded discussion of the interaction potentials and solution regimes can be found in the following: Schmitz, K. S. *Macroions in Solution and Colloidal Suspensions*; VCH Publishers: New York, 1993.

- (15) Stigter, D. *Biopolymers* **1979**, *18*, 3125.
- (16) Belloni, L.; Drifford, M. *J. Phys., Lett.* **1985**, *46*, L1183.
- (17) Schurr, J. M. *Chem. Phys.* **1987**, *111*, 55.
- (18) Akcasu, A. Z. In *Dynamic Light Scattering: The Method and Some Applications*; Wyn Brown, Ed.; Clarendon Press: Oxford, 1993, p 1.

tem give for D_{app} in the DLS experiments

$$D_{\text{app}}(q) = \frac{D_0 H_{\text{pp}}(q)}{S_{\text{pp}}(q)} \quad (3)$$

where D_0 is the infinite dilution value of the diffusion coefficient (self-diffusion), $H_{\text{pp}}(q)$ accounts for hydrodynamic interactions between the polyions, and the static structure factor $S_{\text{pp}}(q)$ is given by

$$S_{\text{pp}}(q) = 1 + 4\pi \langle n_p \rangle \int_0^\infty (\exp[-U_{\text{pp}}(r)/kT] - 1) \exp(-\mathbf{q} \cdot \mathbf{r}) r^2 dr \quad (4)$$

where $q = (4\pi \tilde{n}_0 / \lambda_0) \sin(\theta/2)$ is the scattering vector at angle θ and incident wavelength λ_0 , \tilde{n}_0 is the index of refraction of the solvent, and $U_{\text{pp}}(r)$ is the interparticle pairwise interaction energy. If one employs eq 1 in eqs 3 and 4 then one finds that D_{app} increases as the salt concentration decreases. This behavior is consistent with the o-regime and also the "upper branch" of the systems that exhibit the "splitting" phenomenon, but does not explain the anomalously slow relaxation modes associated with these two phenomena. Furthermore, the decrease in the value of D_{app} with a concomitant decrease in the intensity of scattered light in going from the o-regime to the e-regime is inconsistent with the relationship between these quantities as given by eq 3. The point to be made in this brief discussion is that some polyion systems exhibit a slow mode in the D_{app} profile that cannot be explained in terms of the DLVO theory!

The DLVO repulsive potential is for macroions what the Debye-Hückel potential is for simple salts. Early discussions on the applications and limitations of the Debye-Hückel potential¹⁹⁻²³ also apply to the DLVO potential. The Bjerrum approach²⁰ is to form "ion pairs" such that their respective charges are neutralized and thus removed from the electrical energy. Another approach is based on the statistical mechanical validity of the Debye-Hückel potential and the solution to the Poisson-Boltzmann equation.²¹⁻²³ In the Debye-Hückel theory the mean potential is substituted for the potential of mean force. The error introduced by this approximation is referred to as the "fluctuation potential" from the principles of statistical mechanics.²¹

According to Onsager,²² an interaction energy of the form given by eq 1 is valid only for low concentrations and small charges, where the effects of the ion atmospheres are additive. In the derivation of the DLVO potential,² the macroion concentration is assumed to be sufficiently low that the counterions released by the polyion can be ignored, thus leading to the screening parameter defined by eq 2 and presumably satisfying the additivity requirement for the ion atmospheres. In this regard the standard DLVO potential is a "true" pairwise interaction potential in that it is independent of the concentration

of the macroions. Although its validity is limited to very dilute solutions of macroions and moderate to high ionic strength solvents, the DLVO interaction energy is nonetheless used to interpret data on solutions of finite macroion concentration, and at times salt-free solvents. Generally the results of such an application give a much smaller value for the charge of the macroion than expected from the titration charge.²⁴ These reduced values of the charge are interpreted in terms of "charge renormalization".^{25,26} In other words, the counterions that are associated with the macroion are thermodynamically isolated from those in the bulk solution.²⁶ "Charge renormalization" is thus the macroion analog to the Bjerrum theory of simple salts.

The second approach is to look at the interaction energy itself and effects that go beyond the simple "first approximation" form of the DLVO theory. One such attempt is that of Sogami and Ise,²⁷ who propose a model (hereafter referred to as the SI model) based on the Gibbs free energy. Their Gibbs free energy, $U_{mn}^G(r_{mn})$, has a minimum at separation distances of several thousand angstroms of the particles m and n , which is consistent with the experimental data summarized above. The presence of this long-range minimum has created a controversy in the literature, however.

There is a segment of the scientific community who are of the opinion that Overbeek²⁸ has demonstrated the "error" in the SI theory, which eliminates the long range minimum in $U_{mn}^G(r_{mn})$. To summarize the argument we start with the relationship between $U_{mn}^G(r_{mn})$ and the Helmholtz free energy $U_{mn}^A(r_{mn})$,^{27, 28}

$$U_{mn}^G(r_{mn}) = \left[2 + \kappa'^2 \frac{\partial}{\partial \kappa'^2} \right] U_{mn}^A(r_{mn}) - 2A^\circ \quad (5)$$

$$U_{mn}^A(r_{mn}) = \frac{q_m^{\text{eff}} q_n^{\text{eff}} \exp(-\kappa' r_{mn})}{\epsilon r_{mn}} \quad (6)$$

where κ' includes the released counterions as well as the added electrolyte,

$$\kappa'^2 = 4\pi \lambda_B \left(\sum_{\text{counterion}} Z_j^2 |Z_p| \langle n_p \rangle + \sum_{\text{added}} Z_j^2 \langle n_j \rangle \right) \quad (7)$$

and the effective charge macroion of radius a_m is $q_m^{\text{eff}} = Z_j q_e \sinh(a_m \kappa') / a_m \kappa'$. The factor $\sinh(a_m \kappa') / a_m \kappa'$ results from the assumption of a uniform distribution of surface charge on the macroion. Substitution of eq 6 into eq 5 gives the SI result,

$$U_{mn}^G(r_{mn}) = U_{mn}^A(r_{mn}) \times \left[1 + \frac{a_m \kappa' \coth(a_m \kappa') + a_n \kappa' \coth(a_n \kappa') - \kappa' r_{mn}}{2} \right] \quad (8)$$

To understand the Overbeek approach to the derivation of the correction term,²⁸ it is essential to first

(19) The general discussions on the "Theory of Strong Electrolytes" were held on April 22 and 23, 1927, and these papers are found in *Trans. Faraday Soc.* **1927**, 23.

(20) Bjerrum, N. *Det. Kgl. Danske Videnskab. Selskab Math.-fys Medd.* **1926**, 7.

(21) Fowler, R. H. *Trans. Faraday Soc.* **1927**, 23, 434.

(22) Onsager, L. *Chem. Rev.* **1933**, 13, 73.

(23) Kirkwood, J. G. *J. Chem. Phys.* **1934**, 2, 762.

(24) Shen, E. Y.; Wu, C.-F.; Chen, S.-H.; Blum, L. *Phys. Rev.* **1985**, 89, 3807.

(25) Alexander, S.; Chaikin, P. M.; Grant, P.; Morales, G. J.; Pincus, P.; Hone, D. *J. Chem. Phys.* **1984**, 80, 5776.

(26) Belloni, L. *Chem. Phys.* **1985**, 99, 43.

(27) Sogami, I.; Ise, N. *J. Chem. Phys.* **1984**, 81, 6320.

(28) Overbeek, J. Th. G. *J. Chem. Phys.* **1987**, 87, 4406.

understand the philosophy behind the DLVO theory given by Verwey and Overbeek in terms of the Debye–Hückel charging method.²⁹ The system is partitioned into two regions: “ α ” is that region which contains the macroparticle and the small ions *adsorbed* onto its surface such as to give a charge Z_p , and “ β ” is the remaining solution containing the added electrolyte (denoted “ae”) and the “excess” ions that result from the requirement of charge neutrality for the bulk solution (designated as the counterions and denoted by “c”). In the Verwey and Overbeek treatment²⁸ the chemical work associated with the adsorption of ions onto the surface is *offset by the electrical work in the discharging process*. Therefore one has for this approach the Helmholtz free energy change for the Z_p sites at the surface of the macroion

$$\Delta A_\alpha = \int [d\mu_{\text{chemical}} + d\mu_{\text{electrical}}] Z_p n_p = 0 \quad (9)$$

Verwey and Overbeek² thus conclude that the total free energy of the double layer is equivalent to the work necessary to discharge all of the ions *in solution*, i.e.,

$$\Delta A_\beta = \int n_c d\mu_c + \int n_{ae} d\mu_{ae} = \int n_c d\mu_c \quad (10)$$

In his criticism of the SI theory, Overbeek²⁸ partitions the free energy of the macroions from that of the counterions *in solution*, with the justification that the macroions are fixed in position. The “correction term” introduced by Overbeek is the free energy contribution of the solvent, which is easily obtained from the following relationships:

$$n_s \mu_s = n_s \left(\frac{\partial A}{\partial n_s} \right) = n_s \left(\frac{\partial A}{\partial n_s} \right) \kappa^2 \left(\frac{\partial \kappa^2}{\partial \kappa^2} \right) = -\kappa^2 \left(\frac{\partial A}{\partial \kappa^2} \right) \quad (11)$$

Clearly the inclusion of eq 11 into eq 5 eliminates the term responsible for the second term in the square brackets in eq 8 and results in the relationship

$$0 = n_s \mu_s + n_i \mu_i \quad (12)$$

where the subscript “i” denotes all of the *free ions in solution*.

As reviewed above, it is also quite clear that the Verwey–Overbeek description of the discharging process also omitted the contribution of the solvent. Smalley³⁰ had pointed out that inclusion of the “solvent correction” term into the DLVO theory, where the electrical contribution to the free energy arose solely from the ions *in solution*, resulted in the conclusion that *all electrical interactions would be eliminated!* Overbeek likewise recognized that the small ions did not contribute to the electrical free energy if the solvent contribution was included:²⁸

This remarkable result shows that the solvent and the small ions together just give a zero contribution to G^{el} (and incidentally also to F^{el}).

This conclusion is manifested in eq 12, which is the Gibbs–Duhem result for a system in which the macroions are partitioned from the rest of the solution. One must conclude that if the SI theory is incorrect because of eq 12, then the supporters of the Overbeek

resolution to the minimum in the SI Gibbs free energy must also acknowledge that *the DLVO theory is also incorrect* since inclusion of the solvent term in that derivation likewise eliminates the sole contribution to the electrical free energy, as shown by eq 12. There is no philosophical basis for the selective application of the “Overbeek correction term”: *If it is to be applied to one theory, it must be applied to all theories.*

The resolution to this dilemma is to be found in the inappropriate use of the Gibbs free energy for systems of finite concentrations. In other words, there is a “correction” to the “Overbeek correction” which, when applied, clearly points out the error of selectively isolating the macroions from the rest of the system, and also the physical limitations in the application of eq 11.

It cannot be denied that the macroions and the electrolyte ions as well as the solvent molecules occupy space and therefore contribute to the total measured volume. Since concentrations are measured on the basis of the total volume of the system, the screening parameter must be written as

$$\kappa_i^2 = 4\pi\lambda_B \left(\frac{n_i}{n_s \bar{v}_s + n_m \bar{v}_m + \sum_j n_j \bar{v}_j} \right) \quad (13)$$

where “m” denotes all of the macroions in the total volume of the solution or suspension, the summation is over all of the electrolyte ions present (counterions and added electrolyte), and \bar{v}_n is the volume (in cm^3) of the n th particle type. It is now easily shown that use of the appropriate expressions in eq 11 for the chemical potential and substitution into eq 12 gives the result

$$n_s \mu_s + \sum_j n_j \mu_j = n_s \left(\frac{\partial A_{\text{elec}}}{\partial n_s} \right) + \sum_j n_j \left(\frac{\partial A_{\text{elec}}}{\partial n_j} \right) = \kappa^2 \left(\frac{\partial A_{\text{elec}}}{\partial \kappa^2} \right) [1 - \phi_s - \sum_j \phi_j] \quad (14)$$

where ϕ_j is the volume fraction of the j th ionic species. It is emphasized that eq 14 does not require the macroion to be included in the Gibbs–Duhem expression. Thus the “Overbeek correction” is strictly valid in the *limit of infinite dilution*.

We now include the macroionic species in the integrated form of the Gibbs–Duhem expression, viz.,

$$n_s \mu_s + n_m \mu_m + \sum_j n_j \mu_j = n_s \left(\frac{\partial A_{\text{elec}}}{\partial n_s} \right) + n_m \left(\frac{\partial A_{\text{elec}}}{\partial n_m} \right) + \sum_j n_j \left(\frac{\partial A_{\text{elec}}}{\partial n_j} \right) = \kappa^2 \left(\frac{\partial A_{\text{elec}}}{\partial \kappa^2} \right) [1 - \phi_s - \phi_m - \sum_j \phi_j] = 0 \quad (15)$$

Quite clearly the sum in eq 15 is rigorously 0 when the macroions are included. This result verifies what Ise and co-workers³¹ meant when they criticized Overbeek²⁸ for omitting the macroion contribution in his use of the Gibbs–Duhem expression. It is noted that eq 15 indicates that it is the *free volume* of the

(29) Described on pp 56–60 of ref 2.

(30) Smalley, M. V. *Mol. Phys.* **1990**, *71*, 1251.

solution or suspension that must be used in calculation of the Gibbs free energy. It is thus for this reason that the macroions must be included in the calculation of the total Gibbs free energy, even if the macroions are fixed in position.

What Overbeek had "proven" is not that the inclusion of the solvent *eliminates* electrical interactions but rather that it *verifies the power of the Gibbs–Duhem expression*. The Gibbs–Duhem expression is a statement that the chemical potential changes of all of the species in solution are not independent. Solving for the change in the chemical potential of the solvent, the integrated form of the Gibbs–Duhem expression is

$$\int n_s d\mu_s = -(\int n_m d\mu_m + \sum_j \int n_j d\mu_j) \quad (16)$$

This expression gives the *electrical response of the solvent to the presence of electrically charged solute particles*. The general conclusion given by eq 16 is that the solvent contribution to the Gibbs free energy is given by eq 12 *regardless of the electrical model*.

What may be in question for the various models is the definition of the screening parameter. In addition to the two forms given by eqs 2 and 7 is the one that follows from the coupled mode theories,^{7,11}

$$\kappa^2 = 4\pi\lambda_B \left(\sum_{\text{macroion}} Z_p^2 \langle n_p \rangle + \sum_{\text{counterion}} Z_j^2 |Z_p| \langle n_p \rangle + \sum_{\text{added}} Z_j^2 \langle n_j \rangle \right) \quad (17)$$

Certainly the definition of the screening parameter has a profound effect on the solvent contribution to the free energy as given by eq 17.

It is emphasized that eqs 13–16 do not include a contribution of the macroions to the ionic strength except in the role of a particle of finite size to the volume of the solution or suspension. Since a direct contribution of the macroions to the ionic strength as in eq 17 is a result of mobile macroions, the conclusions drawn from eqs 13–16 are valid for "fixed" macroions as proposed by Overbeek.²⁸

The origin of this "attraction" between the two macroparticles of like charge in the SI model is the presence of excess counterions *between* the two macroions. An analogy is made with the quantum mechanical description of bonding:²⁷

Therefore, at this stage, we should reinterpret U_{mn}^G as the adiabatic pair potential describing the effective interaction between the m th and n th macroions in solution just like the energy eigenvalue of the Schrödinger equation for electronic state in molecule being interpreted as the adiabatic potential between constituent ions.

It is of interest to note that in the model of Ray and Manning³² (hereafter referred to as the RM model) the quantum mechanical analogy is also used in their

description of an attractive component to the Gibbs free energy for parallel rods:

Instead, we find a quantum mechanical analogy in the covalent bond. In their minimum free energy configuration the two lines of charge sites are surrounded by condensed counterions that are delocalized and belong to both, like the valence electrons in a covalent bond between two atoms.

In our own studies we have likewise used the idea of "sharing" of electrolyte ions between *several* macroions as an explanation of the slow mode seen in the D_{app} data,^{33,34} where the attraction between the macroions is described as being due to the following:³⁴

...fluctuations in the distribution of small ions *shared* by several polyions would have a tendency to retard the dissolution of "temporal aggregates."

The picture that emerges from these verbal descriptions is that an "attraction" between macroions of like charge results if electrolyte ions somehow "accumulate and are confined" in a volume of solution that likewise encompasses both macroions. This does not mean, however, that the mathematical descriptions underlying these visualizations are identical. For example, although both the SI and RM models describe the interaction between the macroions in quantum mechanical terms, Manning³⁵ is the first to point out that the RM model is not equivalent to the SI model.

A question of a very fundamental nature raised in the SI theory is whether the Helmholtz free energy or the Gibbs free energy should be used in the description of the system. Quite clearly the *total* volume of the system remains approximately constant during the charging process. However, one must consider the *available* volume for the *thermodynamically* free ions. We use the definition of Oosawa, Imai, and Kagawa³⁸ for the "free volume" as being the total volume minus that occupied by the polyion, the latter being in the present context the macroion and its associated ion cloud or "condensation" volume. That the volume available for the "free" ions changes with ionic strength is part of the Manning condensation model for rods.³⁶ In the Manning model, counterions "condense" in a volume V_c about the rodlike polyion until a certain fraction of charge is neutralized as determined by the linear charge density. Computer simulations by Le Bret and Zimm³⁷ indicate that the value of V_c increases as the ionic strength is lowered; hence the corresponding free volume must likewise decrease even though the total volume remains constant. The RM model for interacting parallel rods further suggests that the "condensation" volume is dynamic in nature as it changes with interparticle distance.³²

When the polyions are moved into the range of intermediate distances, the condensation volume increases dramatically on further approach, the translational entropy of the condensed counterions increases, and approach of the polyions is a spontaneous process. The polyions attract each other.

In other words, as the parallel rods approach each other additional counterions from the bulk solution are transferred to the condensation volume with a con-

(31) Ise, N.; Matsuoka, H.; Ito, K.; Yoshida, H.; Yamanaka, J. *Langmuir* **1990**, *6*, 296.

(32) Ray, J.; Manning, G. S. *Langmuir* **1994**, *10*, 2450.

(33) Schmitz, K. S.; Parthasarathy, N.; Kent, J. C.; Gauntt, J. *Biopolymers* **1982**, *21*, 1365.

(34) Schmitz, K. S.; Lu, M.; Gauntt, J. *J. Chem. Phys.* **1983**, *78*, 5059.

(35) Comment made by Manning during the Polyelectrolyte Symposium at the 205th National Meeting of the American Chemical Society, Denver, CO, March 28–April 2, 1993.

(36) Manning, G. S. *Q. Rev. Biophys.* **1978**, *11*, 179.

(37) Le Bret, M.; Zimm, B. H. *Biopolymers* **1984**, *23*, 287.

(38) Oosawa, F.; Imai, N.; Kagawa, I. *J. Chem. Phys.* **1954**, *22*, 2084.

comitant increase in the magnitude of the condensation volume. Since these "condensed" counterions are thermodynamically isolated from those in the solution, it necessarily follows that the "free volume" available for the "thermodynamically free" counterions must decrease as the ionic strength is lowered. The changes in volume may have a dynamic component, as one may conclude from the calculations of Sánchez-Sánchez and Lozada-Cassou,³⁹ who used a three-point extension of the hypernetted chain (TPE/HNC) approximation to the Ornstein-Zernike equation. In the TPE/HNC calculations the spherical macroions are treated as one component, a dumbbell at fixed separation distance, and two counterions are then "moved" throughout all of the allowed space. The TPE/HNC method therefore includes counterion-counterion correlations. Their computer simulations clearly show that the counterion density about the two spheres increases with an increase in charge density and/or a decrease in the separation distance between the spheres. In the extreme cases of high charge density and small separations, the ion cloud may encompass both spheres such that their identities are lost in the density maps.

There are other mechanisms of electrical origin that give a long-range attractive component to the free energy. Fluctuations in the net charge of the macroion result in an attractive potential of the form given by eq 1, where the square of the magnitude of the charge fluctuations is substituted for the actual charge.⁴⁰ In view of the large differences in the finite sizes of the macroion and supporting electrolyte ions, the "ordering" of the macroions may be entropy driven in a

(39) Sánchez-Sánchez, J. E.; Lozada-Cassou, M. *Chem. Phys. Lett.* **1992**, *190*, 202.

(40) Kirkwood, J. G.; Schumaker, J. B. *Proc. Natl. Acad. Sci. U.S.A.* **1952**, *38*, 863.

(41) Eldridge, M. D.; Madden, P. A.; Frenkel, D. *Mol. Phys.* **1993**, *79*, 105.

(42) Ackerson, B. J. *Nature* **1993**, *365*, 11.

(43) Smalley, M. V.; Schärtl, W.; Hashimoto, T. Submitted.

(44) Comment made in an interview in the NOVA series entitled "The Pleasure of Finding Things Out", broadcast in 1983.

manner similar to that for hard spheres of different sizes.^{41,42} Consider the "void" and "cluster" regions reported by Ise and co-workers.⁵ The entropy of the coions that are repelled from the cluster region is greater for the larger void regions. Smalley, Schärtl, and Hashimoto⁴³ propose a related model in which the clusters are stabilized by the SI potential, and they calculate the "excess" concentration of counterions in the void region. They find that in the region $0.816 < a_p \kappa < 3.05$ there is a maximum in the fractionation of the co-ions that could be as high as 10 times that in the "cluster" region.

There are several experimental data that cannot be interpreted in terms of the classical DLVO potential.³⁻¹⁴ The models discussed herein go beyond the DLVO model in that they include fluctuation contributions to the pairwise interaction between two macroions. The nonuniform distribution of electrolyte ions may therefore play an important role in the explanation of "ordered clumps" of particles by providing an attraction contribution to the free energy of these complex systems. These models are somewhat limited, however, in that only pairwise interactions between the macroions are considered. A complete explanation of the "clumps" must require multiple-body interactions. In the words of Richard Feynman,⁴⁴

We are not trying to check all the time our conclusions. After we checked them enough, they're okay. The thing that doesn't fit is the thing that is most interesting. The part that doesn't go according to what you expected.

I thank Julia P. Fisher for many spirited discussions concerning the preparation of this manuscript.

Kenneth S. Schmitz

*Department of Chemistry
University of Missouri-Kansas City
Kansas City, Missouri 64110*

AR950129R